M2 January 2007

1. A particle of mass 0.8 kg is moving in a straight line on a rough horizontal plane. The speed of the particle is reduced from 15 m s^{-1} to 10 m s^{-1} as the particle moves 20 m. Assuming that the only resistance to motion is the friction between the particle and the plane, find

(a) the work done by friction in reducing the speed of the particle from 15 m s^{-1} to 10 m s^{-1},

(2)

(b) the coefficient of friction between the particle and the plane.

(4)

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2. A car of mass 800 kg is moving at a constant speed of 15 m s^{-1} down a straight road inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{24}$. The resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 900 N.

(a) Find, in kW, the rate of working of the engine of the car.

(4)

When the car is travelling down the road at 15 m s^{-1}, the engine is switched off. The car comes to rest in time T seconds after the engine is switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 900 N.

(b) Find the value of T.

(4)

3.

Figure 1

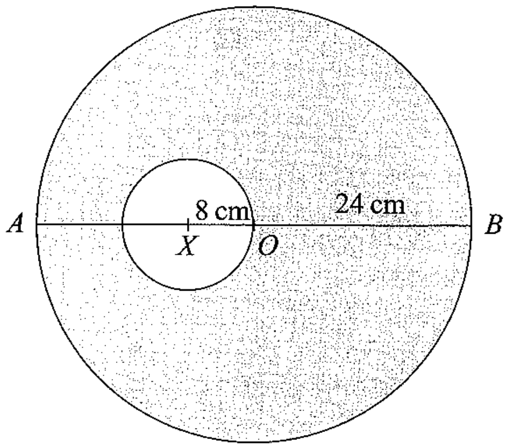


Figure 1 shows a template T made by removing a circular disc, of centre X and radius 8 cm, from a uniform circular lamina, of centre O and radius 24 cm. The point X lies on the diameter AOB of the lamina and $AX = 16$ cm. The centre of mass of T is at the point G.

(a) Find AG.

(6)

The template T is free to rotate about a smooth fixed horizontal axis, perpendicular to the plane of T, which passes through the mid-point of OB. A small stud of mass $\frac{1}{4}m$ is fixed at B, and T and the stud are in equilibrium with AB horizontal. Modelling the stud as a particle,

(b) find the mass of T in terms of m.

(4)

4. A particle P of mass m is moving in a straight line on a smooth horizontal table. Another particle Q of mass km is at rest on the table. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. After the collision, the speed of P is v and the speed of Q is $3v$. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Find, in terms of v only, the speed of P before the collision.

(3)

(b) Find the value of k.

(3)

After being struck by P, the particle Q collides directly with a particle R of mass $11m$ which is at rest on the table. After this second collision, Q and R have the same speed and are moving in opposite directions. Show that

(c) the coefficient of restitution between Q and R is $\frac{3}{4}$,

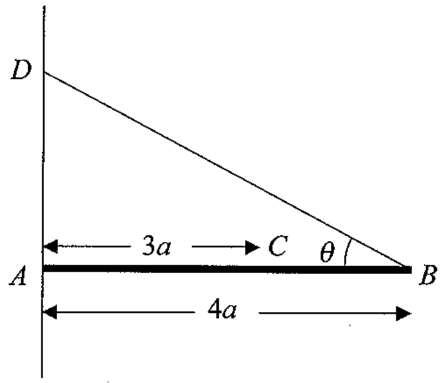
(4)

(d) there will be a further collision between P and Q.

(2)

5.

Figure 2



A horizontal uniform rod AB has mass m and length $4a$. The end A rests against a rough vertical wall. A particle of mass $2m$ is attached to the rod at the point C, where $AC = 3a$. One end of a light inextensible string BD is attached to the rod at B and the other end is attached to the wall at a point D, where D is vertically above A. The rod is in equilibrium in a vertical plane perpendicular to the wall. The string is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$, as shown in Figure 2.

(a) Find the tension in the string.

(5)

(b) Show that the horizontal component of the force exerted by the wall on the rod has magnitude $\frac{8}{3}mg$.

(3)

The coefficient of friction between the wall and the rod is μ. Given that the rod is in limiting equilibrium,

(c) find the value of μ.

(4)

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6. A particle P of mass 0.5 kg is moving under the action of a single force $\mathbf{F}$ newtons. At time t seconds, $\mathbf{F} = (1.5t^2 - 3)\mathbf{i} + 2t\mathbf{j}$. When $t = 2$, the velocity of P is $(-4\mathbf{i} + 5\mathbf{j})$ m s^{-1}.

(a) Find the acceleration of P at time t seconds.

(2)

(b) Show that, when $t = 3$, the velocity of P is $(9\mathbf{i} + 15\mathbf{j})$ m s^{-1}.

(5)

When $t = 3$, the particle P receives an impulse $\mathbf{Q}$ N s. Immediately after the impulse the velocity of P is $(-3\mathbf{i} + 20\mathbf{j})$ m s^{-1}. Find

(c) the magnitude of $\mathbf{Q}$,

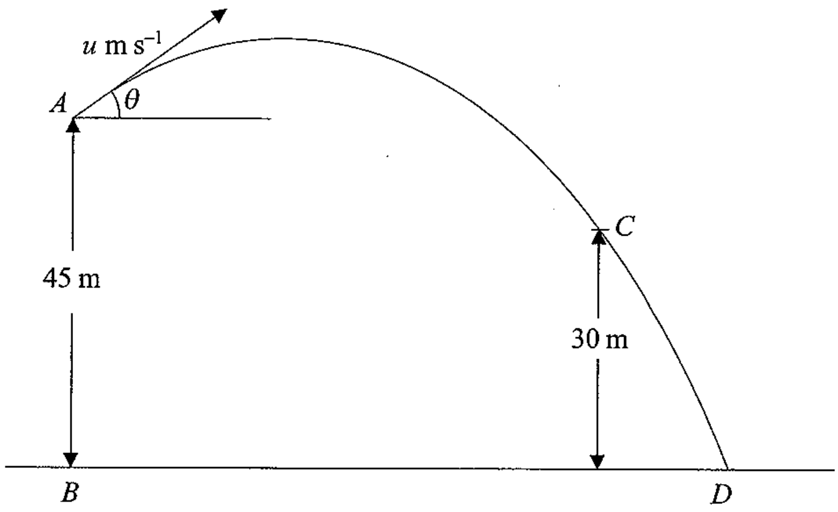
(3)

(d) the angle between $\mathbf{Q}$ and $\mathbf{i}$.

(3)

7.

Figure 3



A particle P is projected from a point A with speed u m s^{-1} at an angle of elevation θ, where $\cos\theta = \frac{4}{5}$. The point B, on horizontal ground, is vertically below A and $AB = 45$ m. After projection, P moves freely under gravity passing through a point C, 30 m above the ground, before striking the ground at the point D, as shown in Figure 3.

Given that P passes through C with speed 24.5 m s^{-1},

(a) using conservation of energy, or otherwise, show that $u = 17.5$,

(4)

(b) find the size of the angle which the velocity of P makes with the horizontal as P passes through C,

(3)

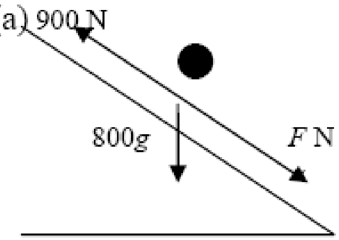
(c) find the distance BD.

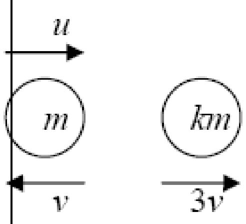
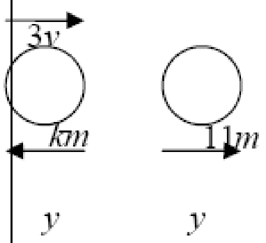
(7)

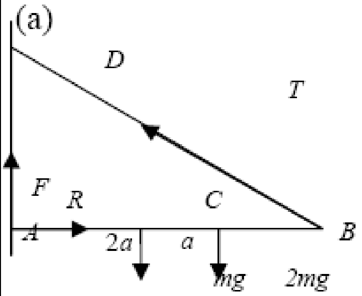
TOTAL FOR PAPER: 75 MARKS

January 2007
6678 Mechanics M2
Mark Scheme

Question Number	Scheme			Marks
1.	(a)	$\frac{1}{2}0.8\left(15^2-10^2\right)=50$ (J)		M1 A1 2
	(b)	$F=\mu R=\mu 0.8g$		M1
	Work-energy	$\mu 0.8g\times 20=50$	ft their (a)	M1 A1ft
		$\mu \approx 0.32$	accept 0.319	A1 4 **6**
	Alternative for (b)			
		$v^2=u^2+2as \Rightarrow a=\frac{15^2-10^2}{2\times 20}=3.125$		M1
	N2L	$F=\mu mg=ma=3.125m$		M1 A1ft
		$\mu \approx 0.32$	accept 0.319	A1 4
	Alternative for (b)			
	WE	$F=\frac{50}{20}\quad(=2.5)$		M1
		$F=\mu R\Rightarrow \frac{50}{20}=\mu 0.8g$	ft their (a)	M1 A1 ft
		$\mu \approx 0.32$		A1 **4**
	The first M1 for *(b)* could be scored in *(a)*:			
		$v^2=u^2+2as\Rightarrow 10^2=15^2-2\times 20\times(-)a\Rightarrow a=(-)\frac{125}{40}$		*(b)*M1
		$F=ma\Rightarrow F=2.5$ $WD=F\times d\Rightarrow 2.5\times 20=50J$		*(a)*M1A1

Question	Scheme	Marks		
2.	(a) 			
	$F + 800g\sin\alpha = 900$	M1		
	$F = 573\frac{1}{3}$	A1		
	$W = 573\frac{1}{3} \times 15 = 8600$	M1		
	$= 8.6$ kW	A1	4	
	NB. Going up hill is an error, not a Misread			
	(b) N2L $800 \times 9.8 \times \frac{1}{24} - 900 = 800a$ *	M1		
	$a = -\frac{43}{60}$ awrt -0.72	A1		
	$0 = 15 - \frac{43}{60}T$	M1		
	$T \approx 21$ accept 20.9	A1cso	4	**8**
	* If they are using their F from (a) then they need to have scored the M1 in (a) in order to score the M1 here.			
	Alternative for (b) WD: $573\frac{1}{3}s = \frac{1}{2} \times 800 \times 15^2$ $s = 157$ Use of $v^2 = u^2 + 2as$ M1 for getting as far as an equation in a. $a = 0.72$ A1 finish as above.			
	2nd Alternative for (b) Ft = Change in momentum: M1 Using the correct F M1 Use of the method to form an equation A1 Equation correct unsimplified but fully substituted A1 $T \approx 21$			

Question Number	Scheme	Marks
3.	(a) Large Small Template	
	Mass Ratios 24^2 8^2, 512 anything in ratio 9 : 1 :8	B1, B1ft
	(c.1810 c.200 c.1610)	
	M(A) $9\times 24 = 16\times 1 + 8\bar{x}$	M1* A1
	$\bar{x} = 25$ (cm) exact	DM1* A1 <u>6</u>
	(b) M(axis) $11M = 12\times\frac{1}{4}m$ ft their $\bar{x}$	M1 † A1ft
	($(36-\bar{x})M = 12\times\frac{1}{4}m$)	
	$M = \frac{3}{11}m$ (o.e.e.)	DM1 † A1 <u>4</u> **10**
4. (a)	 NEL $3v-(-v) = eu$	M1 A1
	$u = 8v$	A1 <u>3</u>
(b)	LM $8mv = -mv + 3kmv$ ft their u	M1 A1ft
	$(m\times(u) = -mv + 3kmv)$	
	$k = 3$	A1 <u>3</u>
(c)	 LM $9mv = -3my + 11my$ ft their k	M1 A1ft
	NEL $2y = e\times 3v$	M1
	$y = \frac{9}{8}v \Rightarrow e = \frac{3}{4}$ ★ cso	A1 <u>4</u>
	(d) $y = \frac{9}{8}v > v \Rightarrow$ further collision between P and Q	M1 A1 <u>2</u> **12**
	A1 is cso – watch out for incorrect statements re. velocity	

Question Number	Scheme	Marks
5.	(a)  M(A) $T\sin\theta \times 4a = mg \times 2a + 2mg \times 3a$ $T = \frac{8mg}{4} \times \frac{5}{3} = \frac{10}{3}mg$ Accept 32.7*m*, 33*m*	M1* A1=A1 DM1* A1 5
	(b) $\rightarrow$ $R = T\cos\theta = \frac{10}{3}mg \times \frac{4}{5}$; $= \frac{8}{3}mg$ ∗ cso ft their T	M1 A1ft; A1 3
	(c) $\uparrow$ $F + T\sin\theta = 3mg \Rightarrow F = mg$ ft their T Or: M(B) $F \times 4a = mg \times 2a + 2mg \times a \Rightarrow F = mg$ $F = \mu R \Rightarrow \mu = \frac{3}{8}$	M1 A1ft M1 A1 4 **12**
	(a) Alternative approach: $\rightarrow$ $R = T\cos\theta$ $\uparrow$ $F + T\sin\theta = 3mg$ M(B) $F \times 4a = mg \times 2a + 2mg \times a (\Rightarrow F = mg)$ $\Rightarrow mg + T\sin\theta = 3mg \Rightarrow T = \frac{2mg}{\sin\theta} = \frac{10mg}{3}$ If they use this method, watch out for F=mg just quoted in (c): M1A1	

6.	(a) N2L $(1.5t^2 - 3)\mathbf{i} + 2t\mathbf{j} = 0.5\mathbf{a}$	M1
	$\mathbf{a} = (3t^2 - 6)\mathbf{i} + 4t\mathbf{j}$	A1 $\underline{2}$
	(b) $\mathbf{v} = (t^3 - 6t)\mathbf{i} + 2t^2\mathbf{j} \quad (+\mathbf{c})$	M1 A1
	$t = 2 \quad -4\mathbf{i} + 5\mathbf{j} = -4\mathbf{i} + 8\mathbf{j} + \mathbf{c} \quad (\mathbf{c} = -3\mathbf{j})$	M1
	$\mathbf{v} = (t^3 - 6t)\mathbf{i} + (2t^2 - 3)\mathbf{j} \quad (\text{m s}^{-1})$	A1
	$t = 3 \quad \mathbf{v} = 9\mathbf{i} + 15\mathbf{j}\ (\text{m s}^{-1})$ * cso	A1 $\underline{5}$
	(c) $\mathbf{Q} = 0.5(-3\mathbf{i} + 20\mathbf{j} - (9\mathbf{i} + 15\mathbf{j})) \quad (= 0.5(-12\mathbf{i} + 5\mathbf{j}))$	M1
	$\lvert\mathbf{Q}\rvert = 0.5\sqrt{(5^2 + 12^2)} = 6.5$	M1 A1 $\underline{3}$
	(d) acute angle is $\arctan\frac{5}{12} \approx 23°$	M1 A1
	or required angle is $\arctan\frac{-5}{12}$	
	or acute angle is $\arccos\frac{12}{13} \approx 23°$	
	or required angle is $\arccos\frac{-12}{13}$	
	required angle is $157°$ awrt $157°$, $203°$	A1 $\underline{3}$ **13**

Question Number	Scheme	Marks
7.	(a) Energy $\frac{1}{2}m\left(24.5^2 - u^2\right) = mg \times 15$	M1 A1=A1
	$u^2 = 24.5^2 - 30g = 306.25$	
	$u = \sqrt{306.25} = 17.5$ ★ cso	A1 <u>4</u>
	(b) $\rightarrow$ $u_x = u\cos\theta = 17.5 \times 0.8 = 14$	B1
	$\psi = \arccos\frac{14}{24.5} \approx 55°$ accept $55.2°$	M1 A1 <u>3</u>
	(0.96 rads, or 0.963 rads)	
	(c) $\uparrow$ $u_y = u\sin\theta = 17.5 \times 0.6 = 10.5$	B1
	$s = ut + \frac{1}{2}at^2 \Rightarrow -45 = 10.5t - 4.9t^2$	M1 A1
	leading to $t = 4.3$, awrt $t = 4.3$ or $t = 4\frac{2}{7}$	A1
	$\rightarrow$ $BD = 14 \times 4\frac{2}{7}$ (14 x t) ft their t	M1 A1ft
	$= 60$ (m) only	A1 <u>7</u> **14**
	Alternative for (a)	
	$\rightarrow u_x = u\cos\theta = 0.8u$, $\uparrow u_y = u\sin\theta = 0.6u$	
	$v_y^2 = 0.36u^2 + 2 \times 9.8 \times 15 = 0.36u^2 + 294$	
	$24.5^2 = u_x^2 + v_y^2 = 0.64u^2, +0.36u^2 + 294$	M1 A1,A1
	$u^2 = 306.25 \Rightarrow u = 17.5$ ★ cso	A1 <u>4</u>
	Alternative for (b) $\rightarrow$ $u_x = u\cos\theta = 17.5 \times 0.8 = 14$	B1
	$\uparrow$ $v_y^2 = u^2\sin^2\theta + 2 \times 9.8 \times 15 = 404.25$	
	$\psi = \arctan\frac{\sqrt{404.25}}{14} \approx 55°$ accept $55.2°$	M1 A1 <u>3</u>
	Alternative for (c) Use of $y = x\tan\theta - \frac{g\sec^2\vartheta}{2u^2}x^2$	M1
	$-45 = \frac{3}{4}x, -\frac{g}{2 \times 17.5^2} \times \frac{25}{16}x^2$	B1,A1
	$x^2 - 30x - 1800 = 0$ o.e.	A1
	Factors or quadratic formula	M1 A1ft
	BD = 60 (m)	A1